Integral of $\dfrac{x^3}{x^2-1}$

Express the integrand as a sum of a polynomial and a proper rational function, then integrate:

$$\int \frac{x^3}{x^2-1}\,dx.$$

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$$\frac{x^3}{x^2-1} = x + \frac{x}{x^2-1}$$

$$x^2-1 \;\overline{)\,x^3}\;\; x$$
$$\underline{x^3 - x}$$
$$x$$

$$= x + \frac{1}{2(x+1)} + \frac{1}{2(x-1)}$$

$$\frac{x}{x^2-1} = \frac{x}{(x+1)(x-1)}$$

$$= \frac{A}{x+1} + \frac{B}{x-1}$$

$$\int \frac{x^3}{x^2-1}\,dx$$

$$x = -1 \Rightarrow A = \frac{-1}{-1-1} = \frac{1}{2}$$

$$= \int x\,dx + \frac{1}{2}\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx$$

$$x = 1 \Rightarrow B = \frac{1}{1+1} = \frac{1}{2}$$

$$= \frac{x^2}{2} + \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C$$

$$\therefore \frac{x}{x^2-1} = \frac{1}{2(x+1)} + \frac{1}{2(x-1)}$$

$$= \frac{x^2}{2} + \frac{1}{2}\ln|(x+1)(x-1)| + C$$

$$= \frac{x^2}{2} + \frac{1}{2}\ln|x^2-1| + C$$